



# CSc355: Section on AI Programming in Scheme

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# Administrative details

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- This part of the course
  - 3 lectures per week for two weeks



# Scheme is based on LISP...

- LISP
  - **LIS**t **P**rocessing language.
    - or, “lots of irrelevant silly parentheses” :-)
  - long pedigree; contemporary with FORTRAN
    - invented by John McCarthy in 1956
  - based on a pure ‘functional’ approach
    - i.e. programs are sets of functions
      - functions return a value and don’t change their arguments
      - no global variables
    - leads to programs that are robust and easy to debug
  - usually interactive and interpreted
  - good for rapid prototyping as well as AI
- Scheme
  - LISP -> Scheme is roughly analogous to C -> Java



# Learning to program in Scheme

- we take a pragmatic rather than an abstract approach
  - a “Scheme 48” interpreter is installed on the PCs in the lab
  - you can get this yourself from <http://www.s48.org/index.html>
  - there’s also an MIT Scheme interpreter on the central University unix server
    - you need to put the following in your .login file:

```
set path=($path /usr/local/packages/mit-scheme-7.3)
set path=($path /usr/local/packages/mit-scheme-7.3/bin)
```
- lots of programming and code examples involved
  - you will learn as much about a ‘different way of programming’ as about AI programming
- you should try things out between the lectures!
  - this is the only way to really understand the material
- exam questions will also be programming/ code oriented



# Outline of what's to come

- core material
  - unit 1: introduction to Scheme: objects and lists
  - unit 2: introduction to Scheme: lists and recursion
  - unit 3: more built-in Scheme procedures
- applying the core material
  - unit 4: search
  - unit 5: more on search
- (5 units in 6 lecture slots allows for some slippage...)



# Books

- “The Little Schemer”, Daniel P. Friedman, Matthias Felleisen, Duane Bibby, January 1996, MIT Press, ISBN: 0262560992
- “The structure and Interpretation of Computer Programs”, Abelson and Sussman (in library)
- Lisp books (for background)
  - “Lisp”, Patrick Henry Winston and Berthold Klaus Paul Horn, Addison Wesley
    - there are many editions, any one is fine
  - “Metamagical Themas: Questing for the Essence of Mind and Pattern”, Douglas Hofstadter
    - contains an excellent short introduction to Lisp



# Unit 1: Introduction to Scheme - objects and lists

- aims
  - to understand basic programming concepts in Scheme
  - to be able to go away and play with the Scheme interpreter



# The Scheme interpreter

- start the interpreter (type “scheme”) and you get a prompt:

>

- Scheme *evaluates* expressions typed at the prompt in a so-called *read, eval, print* loop (REPL)

> ( + 2 2 )

4

>

- can also read in files of expressions—i.e. full programs: use (load “filename.scm”)





# Objects and lists

- Scheme consists of ‘objects’ and ‘lists’
- objects
  - examples: 4 9.6 fred? f-smith “fred” #\a max
  - i.e. ‘symbols’ (including procedure names) and ‘values’
  - special boolean objects: #t and #f
  - no relation to Java ‘objects’!
- lists
  - examples: (a b c) (((a))) ((26 a (b c)) b c 4.1)  
( )
  - i.e. sequences of objects or lists enclosed in brackets

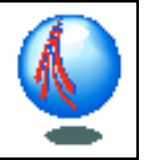


# Evaluating Scheme expressions

- example of an expression

```
> (max (* (+ 6 3) (- 6 3)) 6 (* 2 1))  
27
```

- an expression is a list of sub-exprs or objects
  - the first element must be a symbol that represents a *procedure name* (or a sub-expr that evaluates to a procedure name)
  - the remaining elements are *arguments* to the procedure (or sub-exprs that evaluate to arguments)
- evaluation starts with the arguments
  - symbols evaluate to their values
  - values (numbers, strings etc.) evaluate to themselves
  - evaluation is a recursive process
- then the results are passed to the named procedure
  - procedures can be either built-in or user defined



# Procedures and special forms

- some expressions *look* like procedure calls but are not; they are built-in *special forms*
- these follow individual special rules for evaluation
- examples: quote, if, cond, define, let, begin and lambda...
  - discussion of all of these is coming up...



# A built-in procedure: *reverse*

- if the symbol `x` has previously been given the value of `(a b c)`, then:
  - `> (reverse x)`  
`(c b a)`
  - n.b. `reverse` returns a *new* list, not a changed version of the given list
  - this is generally true; this is why Scheme is a ‘functional’ language
    - but there are a few exceptions...



# A built-in special form: *quote*

- *quote* suppresses evaluation

```
> (quote (a b c))  
(a b c)
```

- i.e. the second sub-expr in the quote expression is *not evaluated*

- usually a quote is expressed in the (entirely equivalent) ‘macro’ form: ``(a b c)`

- e.g.,

```
> (reverse `(a b c))  
(c b a)
```



## More built-in procedures: *car*, *cdr*

*car*: returns a copy of the first element of a list

*cdr*: returns a copy of a list with first element removed

### — examples

(car '(a b c d)) is a

(cdr '(a b c d)) is (b c d)

(car '(((a) b (c)))) is (a)

(cdr '(((a) b (c)))) is (b (c))

(car '(+ 2 2)) is +

(cdr '(+ 2 2)) is (2 2)

(car '()) is ()

(cdr '()) is ()

### — car/cdr applied to a non-list results in an error

```
> (car 6)
```

```
<error>
```



# Extended *car* and *cdr*

- in a slightly bizarre way, *car* and *cdr* generalise to *cXXr*, *cXXXr*, *cXXXXr* etc.

...where each X is either

- an 'a', signifying *car*, or
- a 'd', signifying *cdr*
- evaluation works right to left

`(cadr '(a b c)) = (car (cdr '(a b c))) = b`

`(caar '((a b) c)) = (car (car '((a b) c))) = a`

`(caadr '(a ((b c) d))) = b`

`(cadar '((a (b c)) d)) = b`

etc.



## Another built-in procedure: *cons*

- procedure to add a new element to the front of a list

```
> (cons 'pie '(cake biscuit))
```

```
(pie cake biscuit)
```

```
> (cons 'waffle
```

```
      (cons 'pie
```

```
            (cons 'cake
```

```
                  (cons 'biscuit '())))))
```

```
(waffle pie cake biscuit)
```

- note again that a *new* list is returned; the arguments are not changed...
- ‘consing up a list’





## More built-ins: *if* (special form), *equal?* (predicate procedure)

- value of following is *bus* if  $x=land$ , *ship* otherwise

```
> (if (equal? x 'land) 'bus 'ship)  
ship
```

- note that ‘if’ is a *special form*: only *one* of the two possible result expressions (consequent or alternative) is actually evaluated

- what is the value of the following?

```
(if (equal? (car (reverse '(a b))) 'a) (+ 1 1) 77)
```

- work it out now...



## Another special form: *cond*

- value of the following is *bus* if x=land, or *ship* if x=sea, or *plane* otherwise

```
(cond ((equal? x 'land) 'bus)
      ((equal? x 'sea) 'ship)
      (else 'plane))
```

- can take any number of (predicate result) ‘clauses’
- cond evaluates and returns the result of first clause whose predicate evaluates to #t (or we hit the ‘else’)
  - or it returns #f if no clauses evaluate to #t and there’s no ‘else’ clause
- what is the value of the following?

```
(cond ((equal? (car (reverse '(a b))) 'a) 41)
      (else 77))
```



# The *define* special form

- symbols are given a value (we say: given a “binding”) via the *define* built-in special form
- example

```
> (define bill (if (equal? (car (reverse '(a b))) 'a)
                    (+ 1 1)
                    77))
```

```
> bill
77
```

- note that the first argument (the symbol) is not evaluated



## More built-ins: *begin* (special form), *display* (procedure)

- we can define a list of expressions using the *begin* special form
  - expressions are evaluated sequentially
  - result of the ‘begin’ is the result of the *last* expression; the rest are typically evaluated for side effects (e.g. I/O)
- example:

```
(begin (display "adding 2 and 2") (+ 2 2))
```



## Another special form: *lambda*

- used to define procedures
  - notation derived from lambda calculus
- example:
  - define a procedure that takes one argument:  

```
> (lambda (x) (* x x))
```
  - now apply the procedure:  

```
> ((lambda (x) (* x x)) 2)
```

4
- as above, procedure ‘body’ can be a list of expressions (value of last is returned)
- as seen above, procedures are *anonymous*



# Naming procedures

- we simply bind the value of a procedure to a symbol

- e.g. `(define sqr (lambda (x) (* x x)))`

- more examples

```
(define rac (lambda (l)
```

```
  (car (reverse l))))
```

```
(define rdc (lambda (l)
```

```
  (reverse (cdr (reverse l)))))
```

```
(define how-to-go (lambda (x)
```

```
  (cond ((= x 1) 'land)
```

```
        ((= x 2) 'sea)
```

```
        (else 'air))))
```

n.b. “=” is like “equal?” but does numerical testing

- try defining “snoc”, a backward version of cons

- answer in unit 2



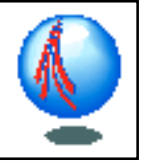
# Naming procedures 2

- a shorthand is available that avoids the use of `lambda`

```
(define rdc (lambda (l)
  (reverse (cdr (reverse l)))))
```

**==**

```
(define (rdc l)
  (reverse (cdr (reverse l))))
```



# Block structure

- the following are equivalent **except that** the *scope* of ‘bill’ is global in the former and local in the latter

```
> (define bill 1)
> (define (fred x) (+ x bill))

> (define (fred x)
  (define bill 1)
  (+ x bill))
```

- better to avoid global scope where possible...





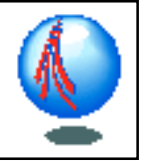
# Summary

- Scheme is an interpreted language and the interpreter evaluates Scheme expressions
- expressions are of the form  
`(procedurename arg1 arg2 ... argn)`
- *procedures* follow standard evaluation rules; but *special forms* don't
- we saw some built-in procedures and special forms
  - `max`, `+`, `-`, `*`, `reverse`, `quote`, `car`, `cdr`, `cXXr`, `cons`, `if`, `cond`, `define`, `begin`, `lambda`, `equal?`, `display`



# Unit 2: Introduction to Scheme - lists and recursion

- aims
  - to understand some more basic Scheme programming concepts - particularly recursion
  - to be able to go away and play more meaningfully with the Scheme interpreter



## Definition of “snoc”

```
> (define snoc (lambda (a l)
  (reverse (cons a (reverse l)))))
```

```
> (snoc 'z '(w x y))
(w x y z)
```



# Repetition

- Jacopini identified *six* programming language facilities required to perform general computation
  - basic operations
    - +, -, \*, /, equal?, car, cdr, cons, ...
  - input/ output
    - read, write, display, ...
  - assignment or binding
    - define, lambda, let...
  - sequencing
    - begin, let and define bodies, ...
  - selection
    - if, cond, ...
  - *repetition*: ??
- repetition in Scheme is usually (not always) achieved using *recursion*



# Example: a “power2” procedure

- the problem...

- define a procedure *power2* that takes an integer argument  $x$  and returns  $2^x$
- so...
  - (power2 0) should return  $2^0 = 1$
  - (power2 1) should return  $2^1 = 2$  *(n.b. 2 times (power2 0))*
  - (power2 2) should return  $2^2 = 4$  *(n.b. 2 times (power2 1))*
  - (power2 3) should return  $2^3 = 8$  *(n.b. 2 times (power2 2))*
  - etc.



# *Power2: the Scheme solution*

- step 1: envisage a (potentially infinite) “tower” of calls with a non-recursive one at the bottom

```
(power2 3) = (* 2 (power2 2))  
(power2 2) = (* 2 (power2 1))  
(power2 1) = (* 2 (power2 0))  
(power2 0) = 1)
```

- step 2: express the procedure in terms of

- i) the simple, non-recursive, case(s); and
- ii) the recursive case(s)

```
(define (power2 x)  
  (if (= x 0)  
      1  
      (* 2 (power2 (- x 1)))))
```

*simple, non-recursive, case*

*recursive case*



# Let's have that in English!

```
(define (power2 x)
  (if (= x 0)
      1
      (* 2 (power2 (- x 1))))))
```

=

```
power2(x)
  if x is 0 then return 1
  else
    return 2 * (power2 with argument x-1)
```

non-recursive case

recursive case



# Isn't this circular?

- we can use Scheme's *trace* procedure...

```
> (trace power2)
> (power2 2)
1. Trace: (POWER2 '2)
  2. Trace: (POWER2 '1)
    3. Trace: (POWER2 '0)
      3. Trace: POWER2 ==> 1
    2. Trace: POWER2 ==> 2
  1. Trace: POWER2 ==> 4
4
```

- so, `power2` is evidently not a circular definition
  - actually its a *linear* definition
  - its linear because it is a tower with a ‘solid foundation’ (i.e. a simple, non-recursive, case)





# Recursive foundations

- a firm foundation
  - man holds photo of himself 10 years younger in which he is holding a photo of himself 10 years younger in which ... of himself as a new born baby; **END**
- some ‘shaky’ foundations
  - box of chocs on which a girl holds a box of chocs on which a girl holds a box of chocs...
  - look in a doubled mirror and see yourself in the mirror...
  - “a horse is a four legged animal that is produced by two other horses...”
  - the numbskulls (little men inside little men...)
  - dictionary defn of recursion: *recursion*: see ‘*recursion*’...



# Example: a pile of stones

- define a procedure that returns a list of  $n$  instances of the symbol **stone** (e.g. (stone stone stone) with  $n=3$ )
- step 1: envisage the tower

```
(pile 3) = (cons 'stone (pile 2))  
(pile 2) = (cons 'stone (pile 1))  
(pile 1) = (cons 'stone (pile 0))  
(pile 0) = '()
```

- step 2: express as i) simple and ii) recursive cases

```
(define (pile n)  
  (if (= n 0)  
      '()  
      (cons 'stone (pile (- n 1))))))
```



# Example: list length

- return the length of a list
- the design
  - simple case: length of empty list is 0
  - recursive case : length of a list is length of (cdr the-list) + 1
- the solution

```
(define (length l)
  (cond ((null? l) 0)
        (else (+ 1 (length (cdr l))))))
```

- (n.b. *null?* is a built-in boolean procedure that returns #t iff its argument is an empty list)



# Example: list membership

- the problem: is a symbol  $s$  present in a list  $l$ ?
- the design:
  - simple case:  $s$  is not present if  $l$  is empty; *or*  $s$  is present if it is equal to the car of  $l$
  - recursive case:  $s$  is present if it is a member of the cdr of  $l$
- the solution

```
(define (member? s l)
  (cond ((null? l) #f)
        ((equal? s (car l)) #t)
        (else (member? s (cdr l)))))
```

- (n.b. there is a built-in ‘member’ that is slightly different)



# Example: summing a list

- the problem: return the sum of a list of ints
- the design:
  - simple case: the sum of an empty list is 0
  - recursive case: the answer is the first number in the list (car) added to the sum of the cdr
- the solution???
- over to you...



## Example: *symbol-count*

- count all the symbols in a potentially nested list
  - note difference from *length*
  - `> '(((a b c) a (a b c)) a ((a b c d) ((a)) a))`  
14
- the design
  - simple case: the argument is an empty list (answer: 0); *or* the argument is not a list at all but a single symbol (answer: 1)
    - (n.b. *symbol?* returns #t iff its argument is an symbol)
  - recursive case: the symbol-count of the car of the list + the symbol-count of the cdr of the list



## *Symbol-count* (cont.)

- the solution

```
(define (symbol-count l)
  (cond ((symbol? l) 1)
        ((null? l) 0)
        (else (+ (symbol-count (car l))
                  (symbol-count (cdr l))))))
```

- n.b.

- (the *first* recursive call on a one-symbol list leads to the ‘symbol?’ case)
- the *second* recursive call on a one-symbol list leads to the ‘null?’ case)



# *Append*: another example with multiple simple cases

- the problem: concatenate two lists

`(append '(a b c) '(d e))` returns `(a b c d e)`

`(append '(a b c) '())` returns `(a b c)`

`(append '(a (b c)) '(d e))` returns `(a (b c) d e)`

- the solution

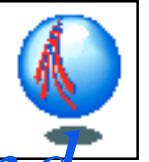
```
(define (append l1 l2)
  (cond ((null? l1) l2)
        ((symbol? l1) (cons l1 l2))
        (else (append (rdc l1)
                        (cons (rac l1) l2))))))
```





## *Append* example

```
(append '(a b c) '(d e)) ->  
(append '(a b) '(c d e)) ->  
(append '(a) '(b c d e)) ->  
(append '() '(a b c d e)) ->  
(a b c d e)
```



## Aside: an alternative formulation of *append* using anonymous functions (*lambda*)

```
(define (append l1 l2)
  (cond ((null? l1) l2)
        ((symbol? l1) (cons l1 l2))
        (else
         (append
          ((lambda (l)(reverse (cdr (reverse l)))) l1)
          (cons ((lambda (l)(car (reverse l))) l1) l2))))))
```

Diagram illustrating the recursive definition of *append* using *lambda* functions:

- The expression `((lambda (l)(reverse (cdr (reverse l)))) l1)` is labeled **'rdc'** (Reverse, Drop, Cons).
- The expression `((lambda (l)(car (reverse l))) l1)` is labeled **'rac'** (Reverse, Access, Cons).



## *Append and list*

- *append* is also available as a built-in procedure
  - takes an arbitrary number of arguments rather than just two
- the built-in procedure *list* is similar to *append* except it makes a list out of its arguments—it doesn't 'run its args together'

```
> (append `(a b) `(c d))  
(a b c d)  
> (append `((a) (b)) `((c) (d)))  
((a) (b) (c) (d))
```

```
> (list `(a b) `(c d))  
((a b) (c d))  
> (list `((a) (b)) `((c) (d)))  
(((a) (b)) ((c) (d)))
```



## Another one for you to try at home...

- define a procedure *flatten* that returns a ‘flat’ list that contains in order all the symbols in its single list argument
- for example

```
> (flatten '((a b c) d (e f g)) h ((i j k l) ((n)) a))
(a b c d e f g h i j k l m n a)
```
- hints
  - there are two simple cases
  - you probably need to use both *list* and *append*
  - not a million miles away from symbol-count! (i.e. work separately on the *car* and *cdr* of the argument)



# Summary

- we've seen how Scheme handles repetition - through *recursion*
- we've studied how to break problems down into recursive solutions
  - simple case(s) and recursive case(s)
  - converging to simple case by (e.g.)
    - decrementing a counter (e.g. pile of stones)
    - 'cdr'ing' down lists (e.g. list length, member)
    - 'car and cdr based recursion' (e.g. flatten, append)
- along the way we've met some more built-ins
  - `symbol?`, `null?`, `append`, `list`, `length`, `member`, `trace`



# Unit 3: More Scheme procedures

- aims
  - to introduce more examples of Scheme code
  - to introduce some new Scheme facilities that will be used in the application-oriented units
    - properties
    - second-order procedures (*sort*, *map* and *apply*)
    - *remove-if*
    - lambda (again)



# First, the homework answer

- problem: define a procedure *flatten* that returns a flat list of all the symbols in its argument

```
> (flatten '((a b c) d (e f g)) h ((i j k l) ((n)) a)))  
(a b c d e f g h i j k l m n a)
```

- solution:

```
(define (flatten s)  
  (cond ((null? s) '())  
        ((symbol? s) (list s))  
        (else (append (flatten (car s))  
                        (flatten (cdr s))))))
```



# Properties

- Scheme symbols may have associated *properties*
  - each symbol can have any number of properties
  - a property is a (*name val*) pair; the choice of both names and values is entirely up to the programmer
- properties are implemented as global ‘2-d table’;  
each symbol’s properties are kept in an list like this:  
`((name1 val1)(name2 val2)...(namei vali))`
- 2d-put! and 2d-get are used to set and get properties

```
> (2d-put! `pyramid-a `colour `red)
> (2d-put! `pyramid-a `is-a `pyramid)
> (2d-get `pyramid-a `colour)
red
```





## *Map and apply*

- map and apply are *second-order procedures*
- *map* ‘iterates’ a procedure over a list of arguments...

```
> (map odd? '(1 2 3))  
(#t #f #t)
```

- (n.b. *odd?* returns #t if its argument is a odd number)

```
(map = '(1 2 3) '(3 2 1)); 2 lists for a 2 arg proc  
(#f #t #f)
```

- note that `(map + '(1 2 3))` produces an error as + needs at least 2 arguments

- to apply a function to a list of arguments we can use *apply*

```
> (apply + '(1 2 3))  
6
```



## *Map and apply (cont.)*

- we can use map and apply to define a faster symbol counting procedure (i.e. one with less recursion)

```
(define (symbol-count1 s); original version
  (cond ((null? s) 0)
        ((symbol? s) 1)
        (else (+ (symbol-count1 (car s))
                  (symbol-count1 (cdr s))))))
```

```
(define (symbol-count2 s); new version
  (cond ((null? s) 0)
        ((symbol? s) 1)
        (else (apply + (map symbol-count2 s)))))
```

- let's check it out with (symbol-count2 '((a b) c)) ...



# Old symbol-count

```
> (symbol-count1 '((a b) c))
```

```
1. Trace: (SYMBOL-COUNT '((A B) C))
2. Trace: (SYMBOL-COUNT '(A B))
3. Trace: (SYMBOL-COUNT 'A)
3. Trace: SYMBOL-COUNT ==> 1
3. Trace: (SYMBOL-COUNT '(B))
4. Trace: (SYMBOL-COUNT 'B)
4. Trace: SYMBOL-COUNT ==> 1
4. Trace: (SYMBOL-COUNT ())
4. Trace: SYMBOL-COUNT ==> 0
3. Trace: SYMBOL-COUNT ==> 1
2. Trace: SYMBOL-COUNT ==> 2
2. Trace: (SYMBOL-COUNT '(C))
3. Trace: (SYMBOL-COUNT 'C)
3. Trace: SYMBOL-COUNT ==> 1
3. Trace: (SYMBOL-COUNT ())
3. Trace: SYMBOL-COUNT ==> 0
2. Trace: SYMBOL-COUNT ==> 1
1. Trace: SYMBOL-COUNT1 ==> 3
```

3



# New version with map/ apply

```
> (symbol-count2 '((a b) c))
```

```
1. Trace: (SYMBOL-COUNT2 '((A B) C))
```

```
2. Trace: (SYMBOL-COUNT2 '(A B))
```

```
3. Trace: (SYMBOL-COUNT2 'A)
```

```
3. Trace: SYMBOL-COUNT2 ==> 1
```

```
3. Trace: (SYMBOL-COUNT2 'B)
```

```
3. Trace: SYMBOL-COUNT2 ==> 1
```

```
2. Trace: SYMBOL-COUNT2 ==> 2
```

```
2. Trace: (SYMBOL-COUNT2 'C)
```

```
2. Trace: SYMBOL-COUNT2 ==> 1
```

```
1. Trace: SYMBOL-COUNT2 ==> 3
```

3

- note that there is significantly less recursion => faster



# Sort

- another simple second-order procedure  
`(sort <sequence> <2-place-predicate>)`
- we can use any 2-place predicate that defines an ordering (i.e. if  $x$  *pred*  $y$  return #t; if  $x$  !*pred*  $y$  return #f)
- examples

```
> (sort '(8 6 3 5 9 2) <)  
(2 3 5 6 8 9)
```

```
> (sort '(8 6 3 5 9 2) >)  
(9 8 6 5 3 2)
```



## *Remove-if*

- a useful second-order procedure that takes a procedure (predicate) and a list, and returns a ‘filtered’ list that is identical except that *the elements for which the predicate is true are omitted*

```
> (define (remove-if pred l)
    (cond ((null? l) l)
          ((pred (car l)) (remove-if pred (cdr l)))
          (else (cons (car l)
                      (remove-if pred (cdr l))))))

> (remove-if fruit?
    '(broccoli milk apple bread butter pear))
(broccoli milk bread butter)
```



# Anonymous procedures revisited

- suppose we want to know which items in a list of groceries are fruit
  - we already know how to do the following:

```
> (define (fruit? x)
    (equal (2d-get x 'kind-of) 'fruit))
```

```
> (map fruit?
    '(broccoli milk apple bread butter pear))
(#f #f #t #f #f #t)
```



# Anonymous procedures (cont.)

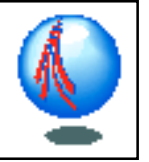
...but it's a pain to have to think up a name and explicitly define *fruit*? when it may only be used once

- *lambda* enables us to conveniently define an ‘anonymous’ procedure on the fly:

```
> (map (lambda (x) (equal (2d-get x `kind-of) `fruit))  
      `(broccoli milk apple bread butter pear))  
(#f #f #t #f #f #t #f)
```

- a lambda expression can go anywhere a procedure name can go (think: lambda == ‘define anonymous’)
- *lambda is especially useful with second order procedures*





# Summary

- we have covered the following:
  - `2d-get`, `2d-put!`, `map`, `apply`, `sort`, `odd?`
- (note a common convention for naming procedures: a ‘?’ suffix denotes a predicate; a ‘!’ suffix denotes a procedure with side effects)
- we now know what a second-order procedure is
  - it’s a procedure that takes a procedure as an argument (or returns a procedure as a result)



# Unit 4: Search

- aims
  - to apply our working knowledge of Scheme
  - to understand the ‘search problem’ (ubiquitous in AI)
  - to develop a *generic* search program that can be specialised for a range of search strategies
  - and then to specialise our generic search program to do *depth first search*
    - we’ll specialise it further in unit 5



# The search problem

- given a problem domain with lots of decision points or potential solutions...
- we *search* through the decision tree or the space of solutions
- examples
  - given a roadmap, find a road route from Lancaster to London
  - find your way out of a maze
  - find the best move in a game such as chess, draughts or noughts-and-crosses
  - find the correct way to parse a sentence
    - “time flies like an arrow”
  - make a goal-directed plan from a set of available sub-plans
    - e.g. make a plan for “*put block A on block B*” from:  
*lift block, move block, clear block, put-down block*



# Representing search

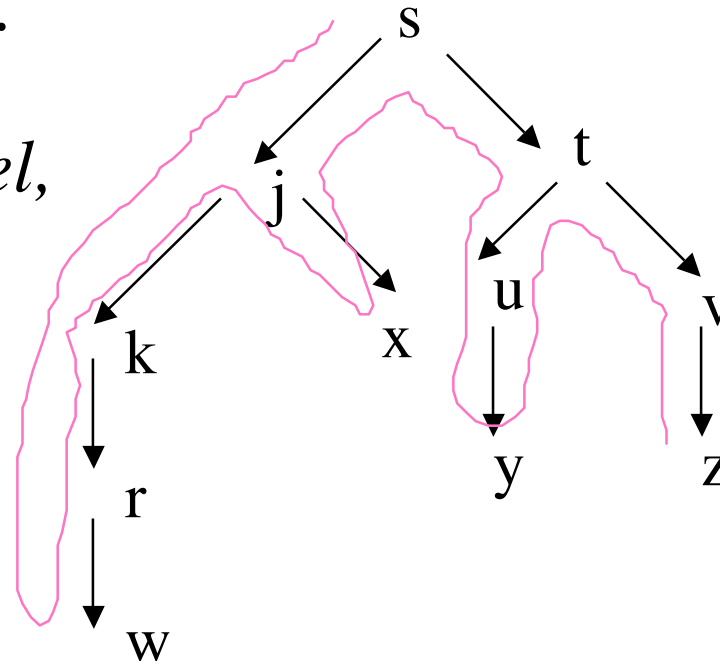
- a search problem is characterised as a *search space* (graph) of *states* (nodes)
  - begin at a *start state* from which there are links to a number of possible *successor* or *child* states (from which there are in turn further successors, etc., etc.)
  - walk the links until we encounter the goal: a *finish state*
- a search space is naturally represented as a graph or a tree
- many *search strategies* are available
  - e.g. depth first search or breadth first search



# Search strategy 1: depth first search

*“Examine **one** child of the start node; if this is not the finish node, examine **one** of its successors (children), again ignoring the rest.*

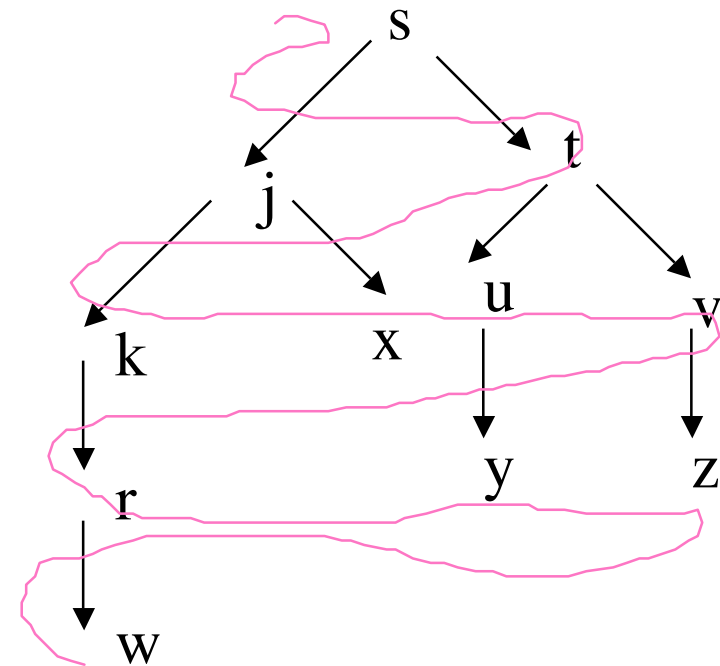
*Do the above repeatedly.  
If there are no links left to explore at a given level, back-up to the last place there was a choice and continue from there.”*





## Search strategy 2: breadth first search

- movement is ‘level by level’:  
*“at each state examine all the children of that state; if we fail to find a finish state among these, move down to one of the children and proceed from there”*
- i.e. the links one level down are fully examined before we go down to the next level





# Representing a search space

– a *tree* can easily be represented as a nested list

- (s (j (k (r (w))) x) (t (u (y)) (v (z))))

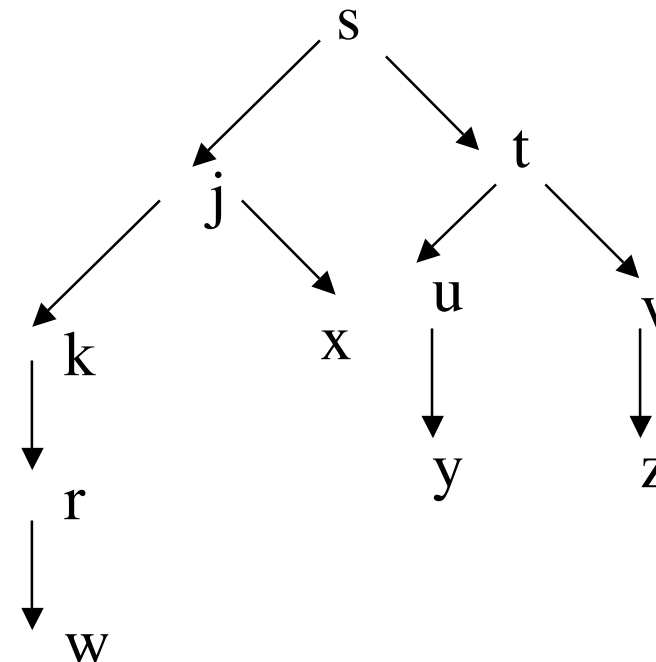
=

(s (j (k (r (w)))

x)

(t (u (y))

(v (z))))





# Depth first search in a nested list (tree)

- basic *dfs* in a nested list is trivial!

```
(define (search finish-state tree)
  (cond ((equal? finish-state tree) #t)
        ((symbol? tree) #f)
        (else (or (search finish-state (car tree))
                    (search finish-state (cdr tree))))))
```

- **but** this approach has severe limitations
  - it only works on trees, not graphs
  - it assumes the tree is already available (i.e., pre-computed)
  - it only returns #t or #f; it doesn't remember the route it took to get to the finish
  - we can't easily extend it to breadth first search and other strategies
- so, we will take a more general approach...





# A better search space representation

- use a ‘children’ property to represent a graph

```
(2d-put! `s `children `(l o))
```

```
(2d-put! `l `children `(m f))
```

```
(2d-put! `m `children `(n))
```

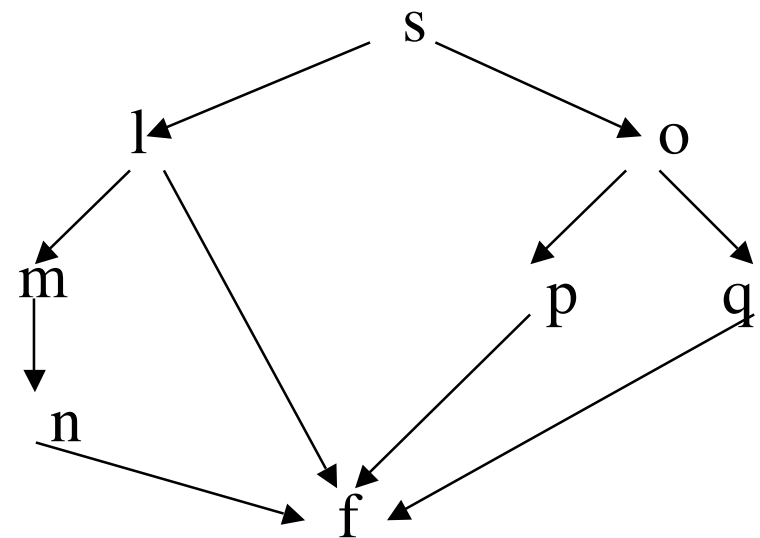
```
(2d-put! `n `children `(f))
```

```
(2d-put! `o `children `(p q))
```

```
(2d-put! `p `children `(f))
```

```
(2d-put! `q `children `(f))
```

- all trees are also graphs, so  
we get trees for free





# A *general* search program (skeleton)

- *search* just calls *search1*, having put the given start state in a list...

```
(define (search s f)
  (define (search1 queue finish)
    (cond ((null? queue) #f) ; not found
          ((equal? (car queue) finish) #t); found
          (else (search1
                  <appropriate merge of
                  (expand (car queue)) and queue>
                  finish))))
  (search1 (list s) f)) ; initialise
```

- ‘queue’ is an ordered list of nodes for us to examine next
- *expand* returns the children of given node



# Specialisation to depth first search

– fill in the blanks as follows

- ‘*expand*’ becomes

```
(define (expand node) (2d-get node `children))
```

- ‘*appropriate merge*’ becomes

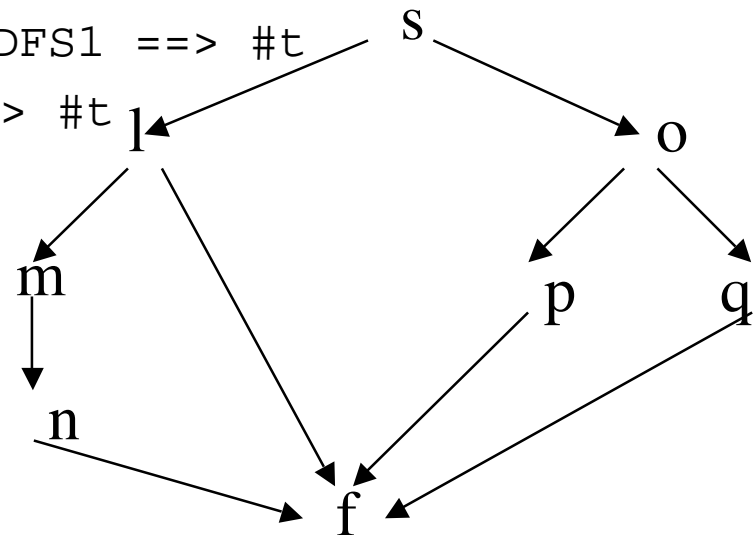
```
(append (expand (car queue)) (cdr queue))
```

```
(define (dfs s f)
  (define (dfs1 queue finish)
    (cond ((null? queue) #f)
          ((equal? (car queue) finish) #t)
          (else (dfs1 (append (expand (car queue))
                           (cdr queue))
                        finish))))
  (dfs1 (list s) f))
```

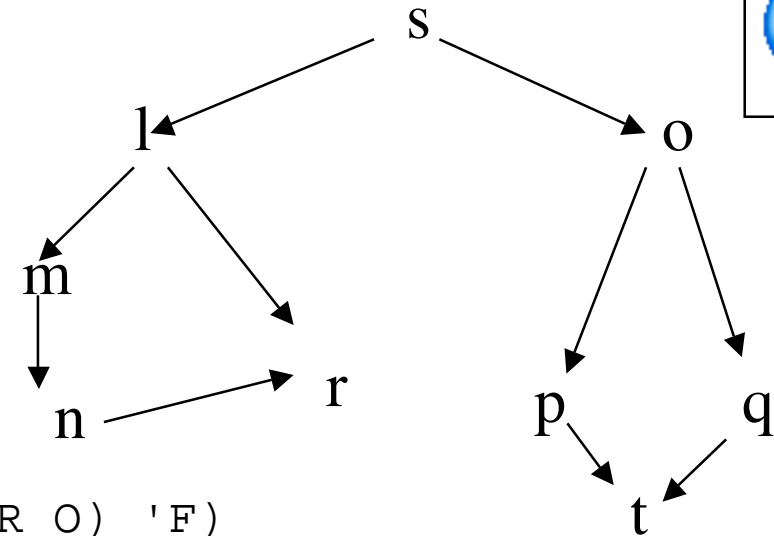


# A trace for a 'town to town' search

```
> (trace dfs dfs1)
(dfs 's 'f)
1. Trace: (DFS 'S 'F)
  2. Trace: (DFS1 '(S) 'F)
    3. Trace: (DFS1 '(L O) 'F)
      4. Trace: (DFS1 '(M F O) 'F)
        5. Trace: (DFS1 '(N F O) 'F)
          6. Trace: (DFS1 '(F F O) 'F)
            6. Trace: DFS1 ==> #t
          5. Trace: DFS1 ==> #t
        4. Trace: DFS1 ==> #t
      3. Trace: DFS1 ==> #t
    2. Trace: DFS1 ==> #t
  1. Trace: DFS ==> #t
#t
```



# Another trace



```
> (dfs 's 'f)
```

```
1. Trace: (DFS 'S 'F)
```

```
2. Trace: (DFS1 '(S) 'F)
```

```
3. Trace: (DFS1 '(L O) 'F)
```

```
4. Trace: (DFS1 '(M R O) 'F)
```

```
5. Trace: (DFS1 '(N R O) 'F)
```

```
6. Trace: (DFS1 '(R R O) 'F)
```

```
7. Trace: (DFS1 '(R O) 'F)
```

```
8. Trace: (DFS1 '(O) 'F)
```

```
9. Trace: (DFS1 '(P Q) 'F)
```

```
10. Trace: (DFS1 '(T Q) 'F)
```

```
11. Trace: (DFS1 '(Q) 'F)
```

```
12. Trace: (DFS1 '(T) 'F)
```

```
13. Trace: (DFS1 '() 'F)
```

```
13. Trace: DFS1 ==> #f
```

(etc.)



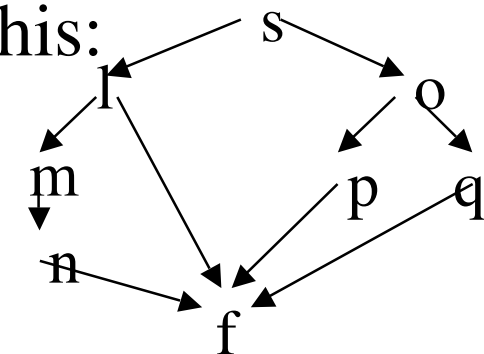
# Making *dfs* return a route

- pack the required information into the queue elements
- the queue *formerly* developed like this (in our first example graph):

(s) -> (l o) -> (m f o) -> (n f o) -> (f f o)

- we will change it so it develops like this:

((s))  
((l s) (o s))  
((m l s) (f l s) (o s))  
((n m l s) (f l s) (o s))  
((f n m l s) (f l s) (o s))



*success achieved when  
(equal? (caar queue) finish))*

*this is the search route* →



# Making *dfs* return a route (cont.)

- new code in bold font

```
(define (dfs s f)
  (define (dfs1 queue finish)
    (cond ((null? queue) #f)
          ((equal? finish (caar queue))
           (reverse (car queue))) ← the returned route
          (else (dfs1 (append (expand (car queue))
                               (cdr queue))
                       finish))))
  (dfs1 (list (list s)) f))

(define (expand route) ; return list of new routes
  (map (lambda (child) (cons child route))
        (2d-get (car route) `children)))
```



# Detecting closed loops in *dfs*

- so far, we have assumed *loop-free* graphs
  - our program will fail to terminate if there are loops!
- we can fix this by augmenting ‘expand’, using *remove-if* to flush already-visited nodes from the route

```
(define (expand route)
  (remove-if (lambda (pth) (member (car pth) (cdr pth)))
    (map (lambda (child) (cons child route))
      (2d-get (car route) 'children))))
```

*our remove-if predicate* →

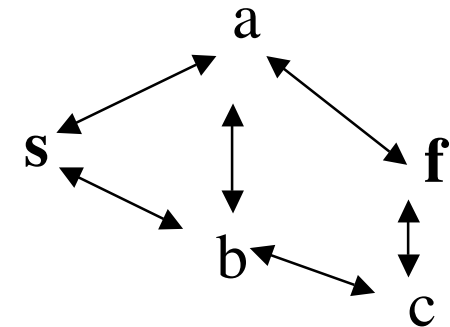




# A *dfs* example with closed loops

– here's the data structure

```
(2d-put! 's 'children '(a b))  
(2d-put! 'a 'children '(s b f))  
(2d-put! 'b 'children '(s a c))  
(2d-put! 'c 'children '(b f))  
(2d-put! 'f 'children '(a c))
```



– given the queue develops as follows:

removed

```
((s))  
((a s) (b s))  
((s a s) (b a s) (f a s) (b s))  
((s b a s) (a b a s) (c b a s) (f a s) (b s))  
((b c b a s) (f c b a s) (f a s) (b s))
```

call: (dfs 's 'f)



# Summary

- search is a problem with states and a state space
- basic tree search is trivial but restrictive;  
a *general* search procedure is more useful
  - can be easily adapted to do *dfs*, *bfs*, ...
  - uses an explicit graph state space representation that is very flexible and extensible (graphs + trees)
  - returns the route traversed
  - detects and avoids loops



# Unit 5: More searching

- aims
  - to continue the exploration of search started in the previous unit
    - breadth first search
    - best first search
    - hill climbing
    - branch and bound
    - beam search



# Breadth first search

- modifying our *dfs* program to do *bfs* is trivial!

just add new routes to the *back* of queue rather than the front

```
(define (bfs s f)
  (define (bfs1 queue finish)
    (cond ((null? queue) #f)
          ((equal? finish (caar queue))
           (reverse (car queue)))
          (else (bfs1 (append (cdr queue)
                               (expand (car queue)))
                       finish)))))
  (bfs1 (list (list s)) f))
```

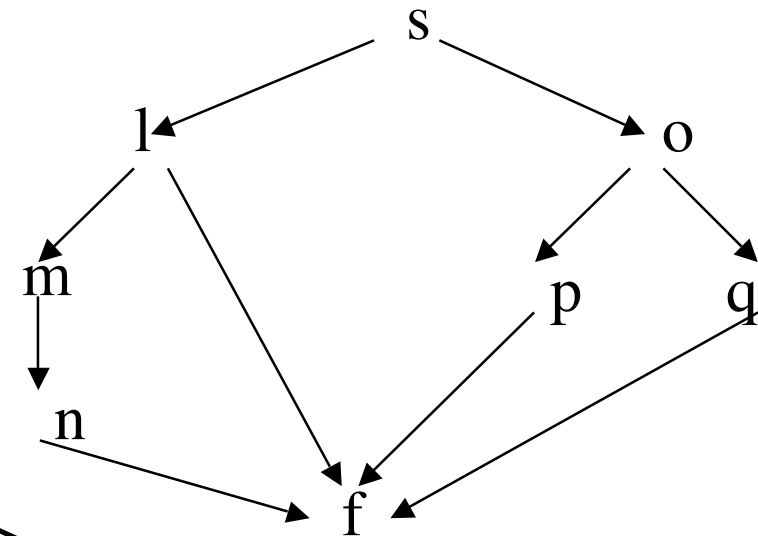
swapped around



# Evolution of the queue of routes in *bfs*

- $((s)) \rightarrow$
- $((l\ s)(o\ s)) \rightarrow$
- $((o\ s)(m\ l\ s)(f\ l\ s)) \rightarrow$
- $((m\ l\ s)(f\ l\ s)(p\ o\ s)(q\ o\ s)) \rightarrow$
- $((f\ l\ s)(p\ o\ s)(q\ o\ s)(n\ m\ l\ s))$

... and on the next expansion we detect success  
(equal? (caar queue) 'f)





# Best first search

- *dfs* expands the *first* route in the queue
- *bfs* expands the *last* route in the queue...
- *best first search* expands the '*best*' route
  - 'best' is that which is estimated to be nearest to a finish node
  - determining 'best' requires a *heuristic* (informed guess)
  - the heuristic sorts the queue of routes according to a predicate *closer*?



# Best first search

```
(define (best s f)
  (define (best1 queue finish)
    (cond ((null? queue) #f)
          ((equal? finish (caar queue))
           (reverse (car queue)))
          (else (best1 (sort (append
                                (expand (car queue))
                                (cdr queue))
                              (lambda (x y)
                                (closer? x y finish)))
                        finish)))))
  (best1 (list (list s)) f))
```



# An implementation of *closer*?

- the following works if we consider ‘as-the-crow-flies’ geographical distance to be a good heuristic
  - we assume properties called X and Y to represent coordinates
  - we assume the existence of a *sqr* square-root procedure

```
(define (closer? a b target)
  (define (distance n1 n2)
    (sqrt (+ (square (- (2d-get n1 'X)
                          (2d-get n2 'X)))
              (square (- (2d-get n1 'Y)
                          (2d-get n2 'Y))))))
  (< (distance (car a) target)
     (distance (car b) target)))
```





# Hill climbing

- like best first search, but, rather than sorting the whole queue, we sort *only the children* of the first queue item and place them at the head of the queue
- hill climbing is thus a compromise between best first search and dfs
  - it minimises best first search's overhead of queue sorting on each expansion
  - while still giving *some* direction to the blindness of DFS
- homework: define a hill climbing variant of our search program



# Branch and bound search

- *guaranteed* to find the ‘shortest’ route from start to finish
- works by sorting the queue of routes in terms of distance travelled so far - always expand the shortest route next
- just use best first search with *shorter?* as sort predicate:

```
(define (shorter? route1 route2)
  (define (route-length p)
    (cond ((null? (cdr p)) 0)
          (else (+ (distance (car p) (cadr p))
                    (route-length (cdr p))))))
  (< (route-length route1) (route-length route2)))
```



# Beam search

- think of someone searching in the dark with a torch with a fixed beam width...
- only keep a *fixed number*,  $w$ , of routes in the queue
  - if there are more than  $w$  routes in the queue discard all but the first  $w$
  - then expand *all* the remaining routes in the queue and sort according to *closer*?
  - *beam search is not guaranteed to find a finish node!*



# Beam search (cont.)

```
(define (beam s f w)
  (define (first-w s w)
    (cond ((zero? w) '())
          (else (cons (car s)
                       (first-w (cdr s) (- w 1))))))
  (define (beam1 queue finish w)
    (cond ((null? queue) #f)
          ((equal? (caar queue) finish)
           (reverse (car queue)))
          (else (beam1 (sort (apply append
                                   (map 'expand
                                        (first-w queue w)))
                               (lambda (x y)
                                  (closer? x y finish)))
                        finish
                        w))))
  (beam1 (list (list s)) f w))
```



# Summary

- we have applied our generalised search program to implement the following:
  - breadth first search
  - best first search - heuristic search
  - hill climbing - gives heuristic direction to DFS
  - branch and bound - guaranteed to find ‘shortest’ route
  - beam search - not guaranteed to find an existing finish
- we have looked at a simple heuristic - distance - which is used by *closer?* and *shorter?*

# Overall conclusion on the Scheme material



- we have learned some Scheme concepts and vocabulary
- we have learned to program using recursion
  - simple and recursive cases
  - converging to simple case by
    - decrementing integer (e.g. pile of stones)
    - cdr'ing down lists (e.g. list length)
    - car and cdr based recursion (e.g. flatten)
- we have looked at search
  - designed a generic search space representation
  - designed a generic search procedure
  - specialised the generic search procedure with various strategies...